Dissociation by acceleration

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We show that mesons, described using rotating relativistic strings in a holographic setup, undergo dissociation when their acceleration a exceeds a value which scales with the angular momentum J as $a_{\max} \sim \sqrt{T_s/J}$, where T_s is the string tension.

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I. INTRODUCTION AND SUMMARY

Rindler horizons provide an intriguing arena in which one can probe aspects of event horizons using simple accelerated probes in flat space-time. One such aspect concerns Fulling-Unruh radiation, which makes accelerated probes feel a thermal bath. While the temperature of this heat bath is relatively low in everyday situations, it has been suggested that large enough accelerations may be achievable in particle accelerators. The idea proposed in [1, 2, 3, 4] is to use polarised *electrons* in a storage ring as thermometers. Their depolarisation rate can then be used as a measure of the local temperature.

In the present paper we want to analyse, instead, the thermal effects of acceleration on *mesons*. In order to describe mesons, we will use generic ideas from holographic models for gauge theories with matter.[1] Large-spin mesons are described in these models by long, rotating strings [6] which end on flavour branes [7]. They bend into the holographic direction, forming a U-like shape. The constituent masses of the quark and anti-quark map to the lengths of the string segments in the holographic direction. In the limit in which the constituent quark masses vanish, mesons are thus simply massless relativistic strings rotating at the position of the "infrared wall" or generalisations thereof. This massless limit is the one we will be considering here.

If we now accelerate this rotating string in one of the three space-like directions, a Rindler horizon forms. The situation is then reminiscent of the analysis of rotating relativistic strings in holographic setups at finite temperature. In those setups, it is known that a horizon at some position in the *holographic* direction leads to the ap-

pearance of a maximum energy and spin, beyond which holographic mesons melt [8]. By analogy with those results, we expect that the acceleration of mesons in the three dimensional space-like directions, and the related Rindler horizon which is now located entirely in the *four-dimensional* world, will also lead to a dissociation effect.[2]

Our results indeed confirm this expectation. We find that spinning strings have an upper bound for their acceleration, which is set by the inverse square root of the angular momentum, $a_{\max} \sim \sqrt{T_s/J}$. As spin-off, we also discuss an extension of this setup, in which the Rindler horizon is used to model aspects of a holographic horizon, obtaining the velocity-dependence of the dissociation length as in [8, 10, 11].

II. ROTATING STRINGS IN RINDLER SPACE

In order to describe accelerated spinning strings, we will use a Rindler coordinate system. A particle which experiences a constant acceleration in the x direction satisfies the equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{m\,\mathrm{d}x/\mathrm{d}t}{\sqrt{1-(\mathrm{d}x/\mathrm{d}t)^2}} = F\,, \tag{1}$$

with $a = F/m$. Solutions to this equation are given by hyperbolas,

$$x^2 - t^2 = a^{-2} \quad \text{i.e.} \quad x = \frac{1}{a}\cosh\left(a\eta\right),\, t = \frac{1}{a}\sinh\left(a\eta\right). \tag{2}$$

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[1] The idea of exploring the thermal effect of acceleration on hadrons has previously been explored by [5], with somewhat similar qualitative conclusions, but their methods are non-holographic in nature and rather different from the ones used in the present paper.

[2] Static Wilson-line configurations in Rindler space-time have been analysed in [9], but these do not exhibit all the phenomena which we find for rotating configurations. More importantly, the main difference of our analysis with respect to [9] is the interpretation of the results. While [9] discusses acceleration in the fifth, holographic direction and relates acceleration to the dissociation temperature in the dual gauge theory, the maximal acceleration discussed here is a genuine four-dimensional one.

To describe accelerated motion, it is convenient to introduce Rindler coordinates (η, ξ) which are adapted to these world-lines, in the sense that fixed-ξ curves map to world-lines of constant acceleration $a = 1/\xi$. More precisely, the coordinate transformation is given by

$$x = \xi \cosh(\kappa\eta)\,, \quad t = \xi \sinh(\kappa\eta)\,. \tag{3}$$

The metric then takes the form

$$ds^2 = -\xi^2\kappa^2 d\eta^2 + d\xi^2 + d\rho^2 + \rho^2\, d\phi^2\,, \tag{4}$$

where we have added an additional two flat dimensions to accommodate the rotating strings which we intend to study. The coordinate system is once more summarised in figure 1.

Our strings will be described by the following ansatz in terms of the world-sheet coordinates (τ, σ),

$$\eta = \eta(\tau)\,, \quad \xi = \xi(\sigma)\,, \quad \rho = \rho(\sigma)\,, \quad \phi = \omega\tau\,. \tag{5}$$

The Nambu-Goto action for our rotating string now reads

$$S = T_s \int d\tau \int_{-L/2}^{L/2} d\sigma \sqrt{\left(\xi^2\kappa^2\dot{\eta}^2 - \omega^2\rho^2\right)\left(\rho'^2 + \xi'^2\right)}\,. \tag{6}$$

We will soon make the gauge choice $\eta = \tau$ which simplifies this action further. The symbol T_s denotes the string tension of QCD string. In the holographic framework, this tension is related to the tension of the fundamental string via the warping factor in the metric in the radial (holographic) direction. Our string will not move freely, following the action (6), as we will also add an external force which acts on the endpoints and accelerates the string. The effect of the force will be described by imposing appropriate boundary conditions in the ξ-direction, as we will explain in more detail below.

The ansatz (5) describes a string which accelerates in the direction *orthogonal* to the rotation plane. It is of course also possible to accelerate the meson in a direction which is under an arbitrary angle with respect to this plane, or even in the plane of rotation. Describing such a string configuration is far more involved as the component of angular momentum orthogonal to the acceleration is not conserved. However, we will argue at the end of the next section that acceleration in this case also leads to dissociation of the meson.[3]

There are two conserved charges which can be used to characterise the string. First, there is the angular momentum J, associated to the rotation in ϕ direction, and second there is the boost charge, associated to the translations in the η-direction. The energy, (associated with translations in the t-direction) is not a conserved quantity, as the boundary conditions applied to the string

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[3] Note that such an acceleration cannot be interpreted as gravitational (i.e. in the spirit of [9]).
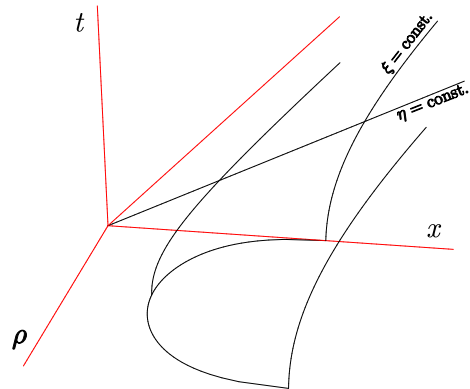

FIG. 1: The coordinate system used to describe rotating accelerated strings. The direction ϕ in which the rotation takes place is suppressed.

endpoints break this symmetry, and lead to a constant increase of the string energy. Explicitly, the angular momentum and boost are given by

$$J = T_s \int_{-L/2}^{J/2} d\sigma\, \frac{\rho^2\, \omega}{X(\sigma)}\,,$$

$$B = T_s \int_{-L/2}^{J/2} d\sigma\, \frac{(-\kappa^2\xi^2)\left(1 + (\xi')^2\right)}{X(\sigma)}\,, \tag{7}$$

where $X(\sigma)$ is defined as

$$X(\sigma) \equiv \sqrt{\frac{\xi^2\kappa^2 - \rho^2\omega^2}{\rho'^2 + \xi'^2}}\,. \tag{8}$$

III. ACCELERATED MESONS

A. Solution for the accelerated meson

The *bulk* equations of motion (i.e. ignoring the surface terms) for the fields ρ and ξ which follow from the action (6) read

$$-\frac{d}{d\sigma}\left(\rho' X(\sigma)\right) - \frac{\rho\omega^2}{X(\sigma)} = 0\,, \tag{9a}$$

$$-\frac{d}{d\sigma}\left(\xi' X(\sigma)\right) + \frac{\xi\kappa^2}{X(\sigma)} = 0\,. \tag{9b}$$

It is easy to check that equations of motion for the other fields (η and ϕ) are automatically satisfied with the ansatz (5).

At the boundaries of the string there are additional terms which need to be taken into account. The external force acts only in the ξ-direction, while there are no forces in the other directions; hence the surface terms for all fields except ξ have to vanish. The surface terms for the fields ρ, ϕ and η are all proportional to $X(\sigma)$, which

allows for two possible boundary conditions,

$$\xi'|_{\sigma=\pm L/2} = \infty\,, \tag{10a}$$

or

$$\rho|_{\sigma=\pm L/2} = \frac{\kappa}{\omega}\xi(\pm L/2)\,. \tag{10b}$$

The surface term for the field ξ is

$$\frac{\delta S}{\delta \xi'}\delta\xi\bigg|_{\sigma=\pm L/2} = \xi' X(\sigma)\delta\xi\bigg|_{\sigma=\pm L/2}\,. \tag{11}$$

We see that if we were to impose the second boundary condition (10b), the boundary term (11) for ξ would vanish automatically, which clearly does not describe an accelerated string with a force applied to its endpoints. Hence, we choose the condition $\xi' = \infty$ which is essentially also the boundary condition imposed on the spinning string solutions of [6]. For this boundary condition, the product of the first two factors in (11) does not vanish. By adding an external force, we impose the Dirichlet boundary condition $\delta\xi = 0$, which forces the endpoints to move with constant acceleration ξ^{-1}.

After choosing the gauge $\rho = \sigma$, the equation of motion (9a) can now be integrated once to give

$$X(\sigma)^2 = -\omega^2\rho^2 + C^2\,, \tag{12}$$

where C is an integration constant. Inserting the definition of $X(\sigma)$ we obtain the differential equation

$$\xi'^2 = \frac{\xi^2\kappa^2 - C^2}{C^2 - \omega^2\sigma^2}\,. \tag{13}$$

This can be integrated to give the solution

$$\xi(\sigma) = \frac{C}{\kappa}\cosh\left[\frac{\kappa}{\omega}\arcsin\left(\frac{\omega\sigma}{C}\right) + D\right]\,, \tag{14}$$

where D is the second integration constant which will set to zero from now on (without any loss of generality, as this just means that we choose the coordinate origin in the σ direction in a symmetric way, i.e. such that $\sigma = 0$ corresponds to the tip of the U-shaped string). This solution also satisfies (9b), and in the limit $\omega \to 0$ it reduces to the solution found in [9].

We now impose the boundary condition, i.e. that $\xi'(\pm L/2) \to \pm\infty$, which fixes the length of the string in terms of the angular frequency and the constant C,

$$\frac{L}{2} = \frac{C}{\omega}\,. \tag{15}$$

Using this relation to eliminate ω from the solution, we find

$$\xi(\sigma) = \frac{C}{\kappa}\cosh\left[\frac{\kappa L}{2C}\arcsin\left(\frac{2\sigma}{L}\right)\right]\,. \tag{16}$$

The total angular momentum for this solution reduces, after elimination of ω, to the simple form
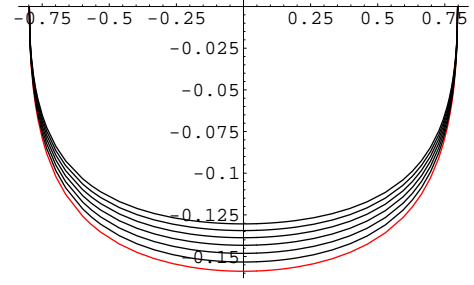
$$J = \frac{\pi}{8}T_s L^2\,, \tag{17}$$

FIG. 2: The shape of the string in the (x,ρ) plane as it evolves in time, in an accelerated frame where the endpoint positions are fixed. The red (bottom) curve represents the shape at $t = 0$.

while the boost charge does not seem to have a simple analytic expression.

In terms of the angular momentum, the shape of the string in the $(\xi, \rho = \sigma)$ plane is

$$\xi(\sigma) = \frac{C}{\kappa}\cosh\left[\frac{\kappa}{C}\sqrt{\frac{2J}{\pi T_s}}\arcsin\left(\sqrt{\frac{\pi T_s}{2J}}\sigma\right)\right]\,. \tag{18}$$

It is more intuitive to describe the shape of the string in the original Minkowski coordinates (x, ρ)

$$x(t,\rho) = \sqrt{\frac{C^2}{\kappa^2}\cosh^2\left[\frac{\kappa}{C}\sqrt{\frac{2J}{\pi T_s}}\arcsin\left(\sqrt{\frac{\pi T_s}{2J}}\rho\right)\right] + t^2}\,. \tag{19}$$

This shape is plotted in figure 2. We see that the solution depends on Minkowski time t, whereas it is stationary with respect to Rindler time η. As time increases, we see that (as expected) all points approach the light-cone $x = \pm t$. We also see that the string configuration (19) does not reduce to a straight, unaccelerated string configuration at the initial time $t = 0$, but that the string is already bent at this moment. This is simply a consequence of the fact that we assume the acceleration to be present at all times; a more realistic solution would start from a straight string, with an acceleration only for $t > 0$, and would exhibit more complicated time dependence, though with qualitatively similar behaviour.

B. Critical acceleration

Let us now study the acceleration of the endpoints and see how it depends on the spin. The constant C in (18) is the acceleration of the midpoint of the string, and related to the acceleration of the string endpoints. Evaluating (18) at $\pm L/2$ we find

$$a^{-1} := \xi(\pm L/2) = \frac{C}{\kappa}\cosh\left[\frac{\kappa}{C}\sqrt{\frac{\pi J}{2T_s}}\right]\,, \tag{20}$$

where we have defined the acceleration of the endpoints a. The acceleration is limited from above by the minimum of the right-hand side of (20) as a function of C. For a generic value of a there are two solutions, one of which is presumably unstable [12]. The maximum of a is achieved for $C/\kappa \approx 0.834\sqrt{\pi J/2T_s}$, resulting in[4]

$$a \leq 0.529\sqrt{\frac{T_s}{J}} \equiv a_{\max} . \qquad (21)$$

For this critical value of the acceleration the string worldsheet is still smooth, and all points move within the light-cone of the string endpoints. However, as we try to increase the acceleration beyond $a_{\max}$, the rigid U-shaped string solution ceases to exist. To determine the precise time evolution of this over-accelerated meson, one would need to consider a more general, time-dependent ansatz. However, a stability analysis of the critical string configuration along the lines of [12] makes it likely that this configuration is unstable, leading to a final point of dissociation given by two disconnected strings stretching all the way to the horizon, and each carrying a fraction of the angular momentum.

From (21) we see that as expected, higher spin mesons are less stable, and dissociate at a smaller value of the acceleration.

To estimate what is the value of the critical acceleration for realistic mesons, we take the value of the string tension to be $T_s = 0.3$ GeV^2. In the holographic framework the description of mesons in terms of large rotating strings is, strictly speaking, only valid for values of angular momentum which are of the order $J \sim \sqrt{\lambda}$, where λ is the 't Hooft coupling. It would be interesting to extend our analysis to the (more realistic) low spin meson sector, using a description in terms of probe-brane fluctuations, but we will present this analysis elsewhere. However, if we naively take the expression (21) and evaluate it for e.g. $J = 1, 2$ and 10, we find for the critical acceleration the values $a_{\max} = 0.290, 0.205$ and 0.092 GeV respectively. It would be interesting to see if there is an experimental set up in which the critical acceleration could be observed. In particular, the decelerations of nucleons in the initial stage of a heavy ion collision is estimated to be of the order of up to a GeV. Although our computation is done for mesons rather than baryons, one may hope that a similar effect (with a similar order of magnitude for the critical acceleration) should hold for baryons as well, and hence may be relevant in describing the dynamics of the initial state of the collision of heavy ions.

As mentioned before, the analysis in the previous section (the ansatz (5)) and the value of the critical acceleration were derived by assuming that the string accelerates in a direction orthogonal to the rotation plane. The

[4] This particular numerical value also plays a role for the computation of [9] which involves static Wilson lines instead of rotating strings.

analysis of the equations of motion describing acceleration under an arbitrary angle is complicated and we will not attempt it here. However, in order to gain insight into what is happening in this case, one can consider a simplified configuration of a stretched string whose endpoints are accelerated in the direction of their relative separation (with the same value of the acceleration). After a finite amount of lab time, the "left" endpoint of the string will cross the Rindler horizon of the "right" endpoint, and hence they will loose causal contact. The lab time of the crossing is given by

$$t_{\mathrm{cross}} = \frac{1}{2}\left(\frac{1}{La^2} - L\right) , \qquad (22)$$

where L is the separation of the string endpoints in the lab frame. There is, trivially, a critical value of the acceleration, a_c, for which the crossing time is zero. If we assume that the separation of the string endpoints originates from rotation, and we use (17), the critical acceleration is $a_c = \sqrt{\pi T_s/8J}$. While the dependence on the string tension is dictated by dimensional arguments, it is interesting that this crude analysis still implies a dependence on the angular momentum as in (21). Hence, both longitudinal and transverse accelerations lead to string melting behaviour.

We should note that in the computation above we have neglected any effects of bremsstrahlung, since in the large-N_c limit in which our holographic computations are valid, the emission of strings is suppressed by the string coupling $g_s \sim 1/N_c$.

Finally, our computation was done in the simplifying limit of vanishing constituent quark mass, although holographic models allow for a description of mesons with non-zero constituent quark masses. It would be interesting to extend our analysis to these more realistic string configurations, and in particular, investigate whether the inclusion of quark masses still leads to a universality of the critical acceleration (21), independent of the supergravity background. It would also be interesting to analyse the combined effect of a Rindler horizon in the four-dimensional space-time and a holographic horizon in the radial direction. We will leave this for future work.

IV. VELOCITY DEPENDENT DISSOCIATION LENGTH

In this section we shift our point of view from the one presented in the previous sections, and use Rindler space (in the spirit of [9]) as a simplified gravitational background, which will allow us to compute the velocity dependence of the dissociation length, as obtained in full-fledged holographic models in [8, 10, 11]. The direction ξ is now interpreted as the holographic, fifth direction, and the acceleration in this direction is caused by the gravitational curvature in the holographic direction, rather

than a genuine four-dimensional acceleration. We generalise the ansatz (5) to

$$\eta = \eta(\tau), \quad \xi = \xi(\sigma), \quad \rho = \rho(\sigma),$$
$$\phi = \omega\tau, \quad y = v\eta(\tau). \quad (23)$$

where (t, ρ, ϕ, y) are directions on our 4-dimensional world, and ξ is the holographic direction. Note that the acceleration (i.e. finite temperature) breaks Lorentz invariance, which makes the boosted solution inequivalent to the unboosted one. Starting from the ansatz (23), the computation of the previous section is trivially extended to yield the string profile

$$\xi(\sigma) = \frac{\sqrt{C^2 + v^2}}{\kappa} \cosh\left[\frac{\kappa}{C}\sqrt{\frac{2J}{\pi T_s}} \arcsin\left(\sqrt{\frac{\pi T_s}{2J}}\sigma\right)\right],$$
$$(24)$$

while the relation between the angular momentum J and the length L of the string in the ρ direction is still given by (17).

The endpoints of the string are now forced to sit on the flavour D-brane, which is located at a fixed position $\xi = \xi_0$. This position is generically a function of the temperature of the background and of the constituent quark mass. On this flavour brane, the local speed of light is given by $c_{\xi_0} = \kappa\xi_0$. The constant C is again related to the position of the midpoint of the string.

In contrast to the situation in the previous section, where we minimised $\xi(L/2)$ as a function of C for fixed J, we thus now want to find the value of C which maximises L for fixed ξ_0. An expression for L is obtained by setting $\xi(L/2) = \xi_0$ in the $v \neq 0$ analogue of (16) and solving for L,

$$L = \frac{4C}{\kappa\pi} \operatorname{arccosh}\left[\frac{\kappa\xi_0}{\sqrt{C^2 + v^2}}\right]. \quad (25)$$

Unfortunately, an analytic solution for the maximum of L for all values of the parameters v and ξ seems out of reach. However, the regime in which $v \to \kappa\xi_0$ can be treated analytically. This regime also corresponds to $C \to 0$, and in this limit the value of C which maximises L is

$$C = \sqrt{\frac{2}{3\,\kappa\xi_0}} v(\kappa^2\xi_0^2 - v^2)^{1/4} \sqrt{\operatorname{arccosh}\frac{\kappa\xi_0}{v}}. \quad (26)$$

Substituting this back into the expression for L and expanding for $v \to \kappa\xi_0$ leads to

$$L_{\max} = \frac{8\sqrt{2}}{3\pi\kappa}(\kappa\xi_0 - v) = \frac{4\sqrt{2}}{3\pi^2 T}(c_{\xi_0} - v) \quad (27)$$

where T is the temperature of the Rindler horizon and c_{ξ_0} is the (local) speed of light. This result indeed fits the full numerical solution for $L_{\max}$ (see figure 3), although in contrast to [8, 10, 11] there is no agreement of this analytic result with a numerical solution in the small-v regime.
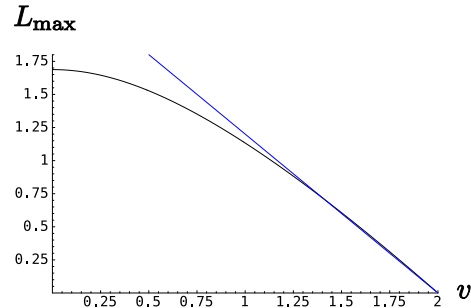

FIG. 3: The numerical solution for the dissociation length as a function of velocity, and the analytic large-v approximation.

Moreover, the form of the expression is different from the one obtained in [8, 10, 11] and we also see that the result does not exhibit Lorentz invariance. The reason for this difference is simple: standard holographic backgrounds at finite temperature all tend to asymptotically flat space, unlike the Rindler background (4) which by construction describes only the near horizon geometry of the non-extremal brane. Asymptotic flatness is a desired property of holographic backgrounds at finite temperature, since it corresponds to the restoration of Lorentz invariance at energies which are much higher than the temperature. The metric (4) and hence the result for the maximal dissociation length (27) by construction do not describe this regime, but they do capture the behaviour of the system at energies which are smaller than or equal to the temperature. In this sense we work in a regime which is complementary to that of [10].

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